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INTERNAL SHOCK WAVES IN SUPERSONIC IDEAL GAS FLOWS PAST WEDGE-PLATE AND CONE-CYLINDER CONFIGURATIONS*

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The problem of internal shock waves (IS) and the reasons for their appearance in inviscid and non-heat-conducting gas flows past wedge-plate (WP) and cone-cylinder (CC) configurations without an angle of attack are studied for the case of an attached bow shock wave and a supersonic flow behind it. For the CC case the axis of the cylinder coincides with the direction of incoming flow. It is shown that, within the limits of validity of the isentropic approximation used in describing flows with weak shocks, the WP is streamlined without IS forming. On the other hand, the sufficient condition for the appearance of IS in the flow modes under discussion is that the flow overexpands directly past the corner representing the point where the generatrices of the wedge (cone) and plate(cylinder) meet. The appearance or non-appearance of such an over-expansion can be established for any intensity of the attached shock wave (AS) in a supersonic flow behind it, without any additional simplifications whatsoever. Computations carried out have shown that the flow past CC contains an IS over the whole range of Mach numbers M_∞ of the incoming flow under investigation. In contrast to this, in the case of WP, flows are possible with overexpansion (the pressure behind the corner is less than the pressure in the incoming flow), as well as with underexpansion, and both situations can be observed, in particular, in the case of a weak AS and low supersonic velocities of the incoming flow, while in the case of the so-called transonic approximation (TA) overexpansion always occurs behind the corner (see /1/)). The lack of overexpansion behind the corner by no means implies that there is no IS for the WP. The fact is that in such cases the change in pressure along the plate can also be non-monotonic. This may happen e.g. if the rarefaction waves emerging from the corner are reflected from the WP, or from its segment, like the compression waves. Recognition of the latter circumstance leads to additional narrowing of the region in which fully supersonic flows past the WP can be realized without IS.

The cases of completely supersonic flows past the WP and CC studied below differ essentially from the cases of mixed flows past the same configurations, when the flow impinging on a wedge or a cone $M<1$ becomes sonic immediately in front of the corner and then disperses in a bundle or rarefaction waves. The latter closes to the right of the IS originating immediately behind the corner, with zero intensity at the corner and tangent to the closing characteristic of the bundle /2–5/. Here the appearance of the IS depends on the structure of the flow near the corner where the Vaglio-Laurin solution holds /6–8/. The necessity for the appearance of IS in the case of WP in the isentropic approximation follows, according to /1/, from a qualitative analysis in the hodograph plane using the Nikolskii-Taganov theorem /9, 10/ on the monotonic nature of the variation in the angle of inclination of the velocity vector $\mathbf{v}$ during the motion along the sonic line.

Thus a flow past a CC always contains an IS. In the case of a WP a fully supersonic flow past it without an IS is possible, although in mixed cases which, basically, occur with a detached bow shock, an IS is obligatory. A similar situation occurs when the plate (cylinder) is replaced by a wedge (cone) with an angle of inclination of the generatrix less then the semivertex angle ϑ_k of the bow wedge/cone). Incidentally, the blunt plate and cylinder can be regarded as a WP or CC with $\vartheta_k=\pi/2$. It should be stressed that IS were first detected experimentally /11/ and computationally /12, 13/ in precisely these cases. Nevertheless, although for the configurations in question under mixed flow modes the appearance of IS follows inevitably from /1–5/, the analysis of the fully supersonic flow of a slightly supersonic stream past a WP carried out within the framework of the TA is often regarded as proof of the necessity of the appearance of IS in all modes of flow with $M_\infty>1$.

Finally, it should be noted that the viscosity and thermal conductivity of the gas may bring substantial corrections into the solution of the problem of IS. Thus, even when there is no IS in an ideal gas, flow with turbulence in the boundary layer past the corner may be

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accompanied by the appearance of an "inverse pressure gradient" /14/ and the associated IS. On the other hand, flow past a fairly acute corner in the ideal approximation (e.g. when $\vartheta_k = \pi/2$) is connected with an appreciable overexpansion of the flow and subsequent sharp increase in the pressure along the plate (the corresponding derivative becomes infinite right behind the corner). In such situations the actual viscosity leads to the formation of a detached wave adjacent to the corner and to the downstream displacement of the IS /11/.

1. The pattern of fully supersonic flow past the WP and CC configurations is shown in Fig.1, where x and y are the axes of a Cartesian or cylindrical coordinate system. Apart from the contour AOB of the body, Fig.1 shows the AS (double line), the c^+-characteristics emerging from the corner, and the c^--characteristics CB. We shall show that in the isentropic approximation, i.e. for AS of "sufficiently low" intensity, in the plane case, all c^+-characteristics are segments of straight lines and the flow to the right of the closing c^+-characteristic OE of the bundle COE is identical with the incoming flow.

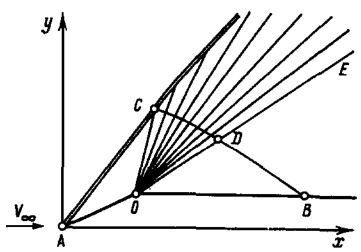

Fig.1

Before doing this we note, that if $z \equiv (p_+ - p_-)/p_-$, where p is the pressure and the minus (plus) index accompanies the parameters in front of (behind) the shock, then, despite the often-held opinion, the condition of sufficiently low intensity of the AS is not at all equivalent to the requirement that $z \ll 1$. Indeed, in the case of a real gas with gas constant R and adiabatic index $\varkappa$, we find that for $z = 0.5$; 1 and 2 the increment $\Delta s \equiv (s_+ - s_-)/(\varkappa R)$ in specific entropy s for $\varkappa = 1.4$ does not exceed 0.005; 0.025 and 0.094 respectively. The values of Δs yield 0.0027; 0.014 and 0.057 in relative increments $\Delta S \equiv (S_+ - S_-)/S_-$ of the entropy function $S = p\rho^{-\varkappa}$ (ρ is the density) and, when $z = 2$ the component of the Mach number in the incoming flow $M_{n\infty} \equiv M_{n-}$ normal to the AS exceeds 1.6, i.e. the shock is not weak. The values of Δs and ΔS given above are computed from the exact relations on the shock. At the same time, the first term of the expansion of Δs in powers of z

$$\Delta s = (\varkappa + 1)\, z^3/(12\varkappa^3) \tag{1.1}$$

increases Δs for the same z by factors of 1.9, 3 and 6.2 respectively. Therefore for (1.1) $z = 0.5$ is already too large.

The following invariants are constant on the characteristics in a supersonic, plane parallel, isoenergetic and isentropic flow:

$$I^\pm = \vartheta \pm \Phi(p), \quad \Phi(p) = \int_{p_*}^{p} \frac{\sqrt{M^2 - 1}}{\rho V^2}\, dp \quad (V = |\mathbf{V}|)$$

where ϑ is the angle of inclination of $\mathbf{V}$ to the axis x; p_* is the critical pressure corresponding to $M = 1$, and I^+ (I^-) is preserved on the c^+ (c^-) characteristics. We know that an invariant which is preserved in the subregions of the continuity of flow on the characteristics intersecting the shock, varies on weak shocks at the same rate as s. Thus in the case of Fig.1 with $z \ll 1$ we have on AS, just as in (1.1) $I_+^- - I_-^- = O(z^3)$. Because of this we find that

$$I^- \equiv \vartheta - \Phi(p) = I_\infty^- = -\Phi(p_\infty) \tag{1.2}$$

within the framework of applicability of the isentropic approximation, and we have a simple wave-type flow with rectilinear c^+-characteristics and uniform flow (identical to the incoming flow) behind the closing characteristic of the bundle. Consequently, if the flow past WP has an AS of low intensity, there is no IS in the flow in the isentropic approximation irrespective of the magnitude of M_∞. It is important to stress that the ranges of smallness of Δs with respect to z and of the validity of (1.2) are very close to each other. We also note that the arguments given here are widely used when analysing steady and unsteady flows with weak shocks, and in particular, in deriving their laws of decay /15/. It is equally well-known that the equations of a shock polar and isentropic compression wave (1.2), or, in the case of a real gas, of the ellipsoid,

$$\vartheta + \alpha + \sqrt{\frac{\varkappa+1}{\varkappa-1}}\, \mathrm{arctg}\left(\sqrt{\frac{\varkappa-1}{\varkappa+1}}\, \mathrm{ctg}\, \alpha\right) - \frac{\pi}{2} = \mathrm{const} \tag{1.3}$$

where α is the Mach angle ($\sin\alpha = 1/M$) are close to each other up to and including $O(z^2)$ for $M_+ \geqslant 1$.

Since the error of the isentropic approximation and the difference between the equations of the shock polar and (1.2) or (1.3) are of the same order, conclusions based on a comparison of these equations and concerning IS whose intensity outside the dependence on M_∞ is found to be of the same order, i.e., $O(z^3)$, are invalid. The same result concerning the outside order of magnitude of the intensity of the IS follows naturally directly from the formulas of TA.

Indeed, let $\eta = (\varkappa + 1)^{1/3}(V - 1)$, where V relates to the critical velocity. Then denoting the parameters in AOC and DOB, by the indices "1" and "2" respectively, we find that

$$\vartheta_1 = (\eta_\infty + \eta_1)^{1/2}(\eta_\infty - \eta_1)/\sqrt{2}, \quad \vartheta_1 + 2\eta_1^{3/2}/3 = 2\eta_2^{3/2}/3 \tag{1.4}$$

within the approximation used /1/. Here the first equation describes the shock wave in TA and the second one describes the bundle of rarefaction waves COE.

Let us introduce the quantities $\varepsilon = (\eta_\infty - \eta_1)/\eta_\infty \geqslant 0$ and $\delta = (\eta_\infty - \eta_2)/\eta_\infty$ characterizing the intensity of AS and the difference in the parameters of the unperturbed flow and the flow in DOB. It can easily be shown that within the approximation used we have $(p_1 - p_\infty)/p_\infty = k\varepsilon$, and $(p_2 - p_\infty)/p_\infty = k\delta$ with $k > 0$. The following relation holds for $\varepsilon \leqslant 1$ and $|\delta| \leqslant 1$ by virtue of (1.4):

$$\delta = -\frac{\varepsilon^3}{96} - \frac{\varepsilon^4}{128} - \frac{11}{2048}\varepsilon^5 + o(\varepsilon^5) + O(\Delta s) \tag{1.5}$$

whose last term characterizes the error of TA.

On the one hand, (1.5) without $O(\Delta s)$ shows, in accordance with /1/, that when the flow past the WP is fully supersonic when $0 \leqslant \eta_1 < \eta_\infty$, and $\varepsilon > 0$, δ is negative, i.e. $p_2 < p_\infty$ and overexpansion of the flow is observed behind the corner within the framework of approximation (1.4), leading to the appearance of IS. On the other hand, since by virtue of (1.5) $p_2 - p_\infty = O(\varepsilon^3) = O(z^3)$ where $z = (p_1 - p_\infty)/p_\infty$, and the entropy increment Δs in the shock to within which the first equation of (1.4) holds, is also of the order of $O(z^3)$, it follows that the magnitude of the overexpansion in question is "of outside order" (it lies outside the limits of validity of the TA). This makes the arguments concerning the necessary appearance of IS to the right of OE, and following from (1.5), invalid.

Fig.2 illustrates the nearness of the shock polar to the epicycloid describing the compression wave at moderate $M_- \equiv M_\infty$ and $M_+ \equiv M_1 \geqslant 1$. We see there, for four values of M_∞, the halves of the exact and "transonic" polar, the epicycloid (1.3) and its analogue from (1.4) where we must write $\vartheta_1 \equiv \vartheta_+ \geqslant 0$ and $\eta_2 \equiv \eta_\infty$ in the second equation. The curve listed above are indicated by the numbers $1, 1', 2$ and $2'$; the solid lines are the polars and the dashed lines are epicycloids ; Fig.2a-d correspond to $M_\infty = 1.1$; 1.3; 1.5 and 1.6 and the angle $\vartheta \equiv \vartheta_+$ is given in degrees. The present and the following results refer to $\varkappa = 1.4$. The passage from $\eta_\pm$ to $p_\pm$ is made for transonic polars and epicycloids using the exact isentropic relations. For all four values of M_∞ the difference between the exact and transonic polars is greater than that between the transonic polars and epicycloids almost everywhere (with the exception, and then only for $M_\infty = 1.1$, of the neighbourhood of the apex of the epicycloid where $M_+ = 1$). Furthermore, the exact polars and epicycloids differ from each other less than the transonic ones and coincide for $M_\infty = 1.5$ within the limits of accuracy of their graphic representation in Fig.2. Finally, for $M_\infty = 1.6$ the exact epicycloid is situated a priori outside the exact polar when the mutual position of their transonic analogues (epicycloid inside the polar) is independent of M_∞. In order to show this fundamental difference for $M_\infty = 1.5$ and 1.6, Fig.2c and d show the curves $1'$ and $2'$, although for such M_∞ the TA is very coarse. At the same time, the same mutual disposition of the transonic and exact polars and epicycloids for M_∞, sufficiently close to unity (Fig.2a and b), shows that despite the fundamental correctness of the conclusions of TA concerning the IS for the case of a fully supersonic flow past a WP, the conclusions may be correct for such M_∞. The latter statement is confirmed by the exact results of the following section.

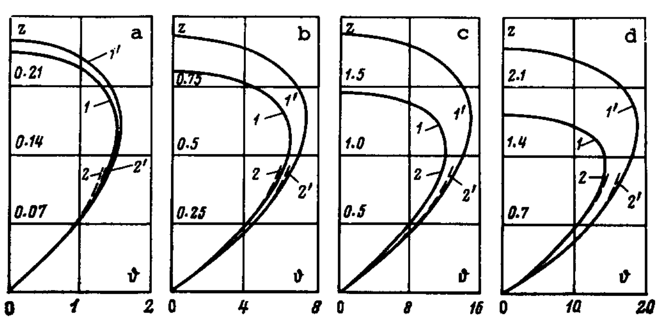

Fig.2

2. When the flow of a real gas past the configurations under discussion is fully supersonic, p_2/p_∞ is a function of M_∞, $\varkappa$ and ϑ_k. The dependence of p_2/p_∞ or $z_2 \equiv (p_2 - p_\infty)/p_\infty$ on the above parameters can be found exactly using the condition that I^- is constant at the

corner point O. If α_k is the Mach angle on the AO, then we can write this condition for the real gas, taking (1.3) into account, in the form

$$f(\alpha_2)=f(\alpha_k)+\vartheta_k,\quad f(\alpha)=\alpha+\sqrt{\frac{\varkappa+1}{\varkappa-1}}\operatorname{arctg}\left(\sqrt{\frac{\varkappa-1}{\varkappa+1}}\operatorname{ctg}\alpha\right) \tag{2.1}$$

All parameters especially α_k and p_k, are known on AO from the solution of the problem of the flow past a wedge or a cone. Therefore, having found α_2 from (2.1), we can easily obtain p_2/p_k in terms of α_k and p_k, and hence $p_2/p_\infty=(p_2/p_k)(p_k/p_\infty)$ and z_2.

The results obtained in this manner for a WP are shown in Fig.3a-c, where the angle ϑ_k is given in degrees; the curves 1,... correspond to different M_∞, every curve either continues outside the graph, or terminates with the angle ϑ_k for which $M_k=1$, i.e. $\alpha_k=\pi/2$. The following correspondence exists between the numbers on the curves and the Mach numbers M_∞ (the brackets contain M_∞): *1* (1.1), *2* (1.2), *3* (1.25), *4* (1.26), *5* (1.27), *6* (1.3), *7* (1.4), *8* (1.5), *9* (1.6), *10* (1.8), *11* (2.0), *12* (2.2), *13* (2.5), *14* (2.75), *15* (3.0), *16* (5.0), *17* (7.5), *18* (10). If we remember that the scale of z_2 in Fig.3a and b differs by a factor of almost 10^4 and in Fig.3b and 3c by a factor of more than 10^2, we see that the smallness z_2 in Fig.3 implies, to even a greater degree than Fig.2, the closeness of the exact equations of the shock polar and epicycloid to each other. This is also implied by the values of $(-z_2)$ corresponding to $M_k=1$. For M_∞ = 1.1; 1.2; 1.3; 1.4; 1.5; 1.6; 1,7; 1.8 and 1.9 they are equal to $4.5\cdot10^{-3}$; $8.7\cdot10^{-3}$; $1.2\cdot10^{-2}$; $1.4\cdot10^{-2}$; $1.5\cdot10^{-2}$; $1.2\cdot10^{-2}$; $9.4\cdot10^{-3}$; $2.3\cdot10^{-3}$ and $6.5\cdot10^{-3}$.

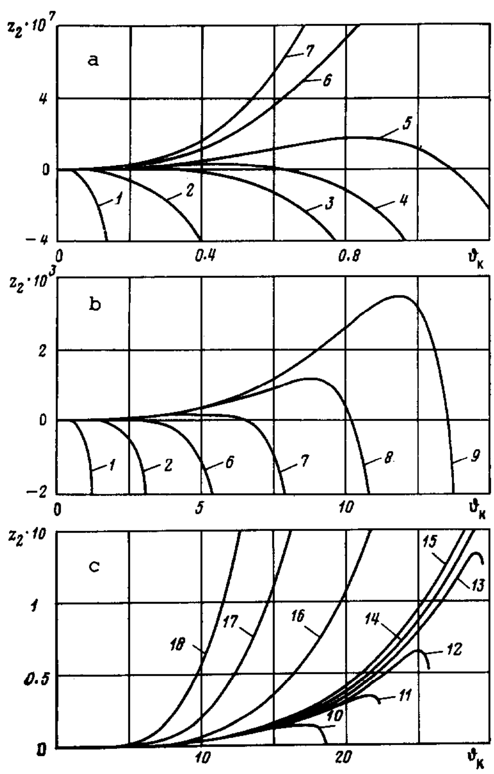

Fig.3

Fig.3 shows that in a flow past WP we have a flow with overexpansion ($z_2<0$, $p_2<p_\infty$), when IS is obligatory, as well as a flow with underexpansion ($z_2>0$, $p_2>p_\infty$). The second possibility becomes prevalent as M_∞ increases. This agrees with the results in /16, 17/ where it was shown that in the case of hypersonic flows past a finite wedge, even the bottom pressure corresponding to $\vartheta_2<0$, exceeds p_∞.

Since, as we move away along the plate to the right $p\to p_\infty$, when $p_2<p_\infty$ retardation of the flow on the plate, according to /1/ and in the light of what was said before, cannot be avoided, and this leads to the appearance of IS. In contrast, when $p_2>p_\infty$, the gas moving along the plate accelerates "in the mean". However, even then the change in pressure can be non-monotonic. This is mainly caused by the form of the reflection of the rarefaction waves forming the bundle COE, from the WP (in the isentropic approximation discussed above this reflection does not occur). We know (e.g. /18/) that at every M_∞ or V_∞ the sign of λ, which is the reflection coefficient of the perturbations arriving at the shock along the c^+-characteristics depend on the angle of inclination of the shock σ. The possibilities arising in this case are shown in Fig.4. Here, in the $V_\infty \sigma$ plane where V_∞ is referred to the cirtical velocity, the region contained between the upper dashed line corresponding to the speed of sound behind the shock and the lower solid line representing the characteristic $\sigma = \arcsin (1/M_\infty)$, corresponds to the supersonic velocities of the flow behind the shock. On the characteristic $\lambda = 0$ and on the other two solid lines, which unlike the characteristic, yield "non-trivial" solutions of the equation, $\lambda(V_\infty, \sigma) = 0$.

In the regions I and III $\lambda < 0$ the rarefaction waves as well as the compression waves are both reflected from the shock. In the regions II and II' with a common boundary along a segment of the vertical shown in Fig.4 with dashed line, $\lambda > 0$. For V_∞ and σ from II and II' the rarefaction waves are reflected from the shock as the rarefaction waves. Point C in Fig.1 has a corresponding point c in Fig.4, belonging to one of the regions I, II, II' or III. For the whole AS to the right of C we have in Fig.4 the corresponding segment of the vertical, connecting c with the lower solid curve, i.e. with the c^+-characteristic into which AS degenerates at infinity. For the points c from II the rarefaction waves of the bundle are reflected from the whole AS as rarefaction waves. In all other cases at least part of the rarefaction waves is reflected from the AS as compression waves. In such situations the IS may, in principle, appear already as the result of intersection of the reflected c^-- characteristics. However, if this does not occur, then the compression waves from the infinitely long c^+-characteristics will, after reflection from the plate, necessarily intersect. Therefore, already when $p_s > p_\infty$, supersonic flow past WP without IS is possible only for the point c belonging to II. We note, by the way, that for weak AS the possibility of underexpansion of the flow ($p_s > p_\infty$ or $z_s > 0$) shows the importance of the term $O(\Delta s)$ in (1.5). Indeed, we can show that for small δ and Δs

$$z_s = \frac{\varkappa (V_\infty - 1) M_\infty^2}{V_\infty} \delta - \frac{\Delta s}{R}$$

where, as before, $\delta = (\eta_\infty - \eta_s)/\eta_\infty = (V_\infty - V_s)/(V_\infty - 1)$. Hence, it follows from (1.5) that without the term $O(\Delta s)$ in (1.5) z_s will always be negative. The latter contradicts the exact results shown in Fig.3.

Computations analogous to those carried out for a WP, were performed for a CC. They showed that in the case of fully supersonic flow past a sharp cylinder, for M_∞ and ϑ_k from the range of values overlapping the range of values of the parameters from the tables of flows past circular cones /19/, an overexpansion of the flow ($z_s < 0, p_s < p_\infty$) takes place behind the corner. Moreover, the derivative $(dp/dx)_s$ along OB at the point O, computed using the method and results of /20/ for $\vartheta_k = 10, 15, 20$ and 25° at $M_\infty = 1.6$; $2.0; 2.4; 3.0; 4.1$ and 5.2, and for $\vartheta_k = 10^\circ$ also for $M_\infty = 1.2$, was positive in all cases. Thus here, unlike the WP where a small flow occurs in DOB, the retardation of the gas begins right behind the corner. Therefore, the supersonic flow past a sharp-edged cylinder is accompanied, at least in all cases studied here, by the appearance of IS.

As we have already noted, the analysis carried out can also be applied to configurations obtained by replacing the plate (cylinder) by a wedge (cone) with positive angle of inclination of the generatrix $\vartheta_s < \vartheta_k$. In particular, the computations carried out using the results obtained in /20/ for the values of ϑ_k and M_∞ listed above, showed the following. In all cases discussed here, except $M_\infty = 4.1$ at $\vartheta_k = 25^\circ$ and $17^\circ \leqslant \vartheta_s < 25^\circ$ and of $M_\infty = 5.2$ for $\vartheta_k = 20^\circ$ and $11^\circ \leqslant \vartheta_s < 20^\circ$, as well as for $\vartheta_k = 25^\circ$ and $10^\circ \leqslant \vartheta_s < 25^\circ$, the derivative $(dp/dx)_s$ is positive (of course $(dp/dx)_s = 0$ when $\vartheta_s = \vartheta_k$). On the other hand, when $\vartheta_s > 0$, the conditions of the appearance of IS are less favourable in both the plane and axisymmetric case, for two reasons. First, when $\vartheta_s > 0$ it is the finite length of the c^+- characteristics emerging from the body (all these characteristics, unlike the case $\vartheta_s = 0$, arrive at the AS). Secondly, since AS does not degenerate into a characteristic when $\vartheta_s > 0$, it follows that the reflection of the rarefaction waves from the AS without change in sign, can become possible, beginning with some ϑ_s, also for the points c of region II' in Fig.4. The circumstances noted above extended the range of parameters for which flow without IS occurs for the configurations in question. In the limiting case of $\vartheta_s = \vartheta_k$, we obtain trivial cases (flow past an infinite wedge and a cone with attached shock wave) of flows without IS. In spite of those trivial and non-trivial cases discussed above (first of all that of WP with point c from II), the analysis carried out confirms, in agreement with the results of /1–5, 1–14/, that supersonic flows in semibounded regions not containing the IS, are more the exception than the rule. The latter is natural for the solutions of quasilinear, hyperbolic-type equations.

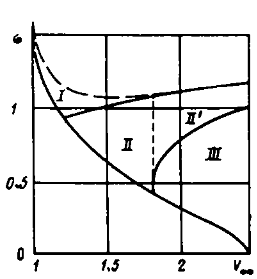

Fig.4

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